# Analysis of Uses of Technology in the Learning of Mathematics

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## Introduction

In ICME12, the role of technology in mathematics education was divided into two distinct study groups: Analysis of uses of technology in the teaching (TSG 18) and learning (TSG 19) of mathematics. Of course, these two aspects of mathematics education are closely intertwined, but we tried to concentrate the TSG 19 discussions around the aspect of LEARNING with ICT (Information and Communications Technology).

The TSG 19 especially addressed the following issues in the learning of mathematics:

- the design of digital technology
- the design of learning environments
- large-scale and long-standing digital technology implementation projects
- assessing mathematics learning with and through digital technologies
- the interaction between ICT and learners of mathematics
- connectivity of ICT
- theoretical and empirical models for learning with ICT
- the implementation of curricula

**Organizers** Co-chairs: Marcelo C. Borba (Brazil), Hans-Georg Weigand (Germany); Team Members: Ornella Robutti (Italy), Mónica Villarreal (Argentina), Tom Dick (USA), Youngcook Jun (Korea); Liaison IPC Member: Yuriko Baldin (Brazil).

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## Outline of Contributions

All submitted papers were reviewed by three reviewers and 33 papers and one poster were finally accepted. For presentation, papers were grouped into four groups:

- Group A: E-learning, Interactive Textbooks, Games, Mobile Applications
- Group B: Theoretical Aspects
- Group C: Dynamic Geometry Systems (DGS), Calculators, CAS
- Group D: Topics in Mathematics

Each of the four 90-min sessions was devoted to one of these four groups of papers. The time available did not allow for formal presentations of every paper by their authors. Two papers from each group were selected for presentations by the authors. The remaining papers in that group were summarized by a member of the organizing committee, with opportunities for comments by the authors and for discussion of the papers by all participants. The structure for each 90-min session included some brief opening remarks by the co-chairs of the committee, followed by a 30-min period for summary and discussion of those papers not presented later in the session. Following this summary discussion, each of the two selected papers were presented by their authors (15 min each, with 10 min for presentation and 5 min for discussion). After the individual paper presentations, participants engaged in 15 min of roundtable discussions focused around questions of emergent issues raised by the papers considered in that session. At the conclusion of each session, the TSG 19 co-chairs had made some brief closing remarks.

## Group A: E-learning, Interactive Textbooks, Games, Mobile Applications

- Gerry Stahl (College of Information Science, Drexel University, Philadelphia, USA): Designing a Learning Environment to Promote Math Discourse
- Robyn Jorgensen (Griffith University—Australia), Tim Lowrie (Charles Sturt University—Australia): Digital Games and Mathematical Learning: A summary paper

Gerry Stahl emphasized the fact that more and more teachers and students were learning online—with distance education, online masters programs, home schooling, online high schools, etc.—which makes the incorporation of virtual collaborative learning environments a natural trend. He presented a virtual GeoGebra learning environment that integrates synchronous and asynchronous media with an innovative multi-user version of a dynamic math visualization and exploration toolbox.

Jorgensen and Lowrie presented a summary of a three-year project that explored the possibilities of digital games to enhance mathematical learning. They especially found that using games in classrooms might have much more benefits than just learning mathematics.

## Group B: Theoretical Aspects

- Abramovich Sergei (*State University of New York at Potsdam, USA*), Eun Kyeong Cho (*University of New Hampshire, USA*): Pre-teachers' learning of mathematics through technology-enabled problem posing
- Barbara Schmidt-Thieme (University of Hildesheim Germany), Hans-Georg Weigand (University of Wuerzburg, Germany): Choosing adequate Digital Representations,

Abramovich and Cho considered the potential of new technologies to turn a routine arithmetical problem into a challenging mathematical investigation. The authors suggested that an important didactic task for teachers will be to decide if technology-enabled problem posing results in a contextually, numerically, and pedagogically coherent problem. This influences the choice of the adequate software.

Schmidt-Thieme and Weigand presented examples of students' working with representations and posed some main future research questions concerning the use of representations in a technology-based environment, e.g.: Which criteria characterize an adequate representation of a problem's solution? Which different levels of argumentation, reasoning and proof are related to a special representation? Which criteria characterize a good (in the sense of giving some feedback about learners' competencies) documentation of a solution of a problem?

## Group C: Dynamic Geometry Systems (DGS), Calculators, CAS

- Arthur B. Powell, Loretta Dicker (Rutgers University, USA): Toward Collaborative Learning with Dynamic Geometry Environments
- Thomas Lingefjärd, Jonaki Ghosh, Aaloka Kanhere (Technology Working Group of the Indo Swedish Initiative in Mathematics Education): Students Solving Investigatory Problems with GeoGebra—A Study of Students' Work in India and Sweden,

Powell and Dicker presented a model of collaborative, online learning with a dynamic geometry environment that supports collaboration around mathematical problem solving and development of significant mathematical discourse. The

authors especially intend to motivate in-service secondary teachers in designing curricular units that develop students' significant mathematical discourse as they develop geometric ideas.

Lingefjärd, Ghosh and Kanhere started with the hypothesis that the use of technology in mathematics instruction might lead from an experimental mathematics, that is, verification and conjecturing, to theoretical mathematics, that is, formal abstract concepts and proofs. The authors had done a parallel experimental study in Sweden and India using a dynamical geometry environment and getting quite similar results concerning the working styles of students in these two countries.

## Group D: Topics in Mathematics

- Christian Bokhove (St. Michaël College, Zaandam, the Netherlands/Freudenthal Institute, Utrecht University, Utrecht, the Netherlands), Paul Drijvers (Freudenthal Institute, Utrecht University, Utrecht, the Netherlands): Effects Of A Digital Intervention On The Development Of Algebraic Expertise
- Jens Jesberg, Matthias Ludwig (Goethe University Frankfurt, Germany): MathCityMap—Make mathematical experiences in out-of-school activities using mobile technology

Bokhove and Drijvers especially wanted to answer the question about the effect of an intervention, consisting especially of diagnostic digital modules, on the development of algebraic expertise, including both procedural skills and symbol sense. They observed "a large effect on improving algebraic expertise" after an intervention of just 5 h.

Jesberg and Ludwig presented a "MathCityMap-project", which is based on a GPS technology. High school students experienced mathematics at real locations and in real situations within out-of-school activities, with the help of GPS-enabled smartphones and special math problems.

## Conclusions

More than thirty years have passed since the first ICMI study group on technology. Papers presented in this TSG show that the work with technologies can present new trends even though one can no longer refer to digital technologies as "new technologies". Digital tablets and devices that increasingly enhance the possible interactions between humans and technology were presented as means for transforming the way students can know. Many of these devices imply changes in curriculum and challenge the structure of time in school. In other words, if they are to be used in school, students will either have to be outside class using mobile

technology, or in class using them for longer periods of time. TSG 19 was diverse enough that many papers also proposed how technology can be used now, without many changes in the way school is organized. “Geogebra” is one of those key applications used at this conference. The free software seems to have found many different followers in different countries and it has been used in different manners. Some have incorporated it into online learning environments, while others are developing ways of annotating the screen of Geogebra.

Last but not least, findings of new technological developments and of research results were discussed in small groups, overcoming language barriers. The situation is the same in mathematics classrooms all over the world. Apart from special and valuable cultural divergence and distinctions new technologies reveal the same or at least similar problems in mathematics learning all over the world and they may be a catalyst to forward important developments in mathematics classroom activity.